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The anisotropy of ultra-high-energy cosmic rays I. Large-scale features

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Abstract. Until a few months ago there seemed to be no evidence for any large-scale anisotropy in the arrival directions of ultra-high-energy ($E > 10^{17}$ eV) cosmic ray primaries. In a recent letter, Krasilnikov *et al*, however, have put forward some evidence for a huge anisotropy above 10^{19} eV, and this evidence has been claimed to be supported by some earlier results at lower energies. The present paper gives a detailed discussion of the statistical selection effects involved and concludes that the anisotropy is not proven. Some astrophysical implications of the anisotropy, if genuine, are also considered.

1. Introduction

One of the most intriguing features of cosmic ray physics is the high degree of isotropy of the arrival directions of the primaries throughout the energy range where interplanetary effects are thought to be negligible (ie above a few hundred GeV for protons). The lack of marked anisotropies might be due to the combined effect of source distribution and propagation. Below 10^{17} eV, all plausible source candidates are several Larmor radii away and the particles are expected to undergo several random scatterings before reaching the earth. In such diffusive-type situations low-order spherical harmonics will dominate the angular distribution and greatly reduce the information about the individual sources.

In a 3 μG interstellar field (a typical value in practice) the Larmor radius (r_L) for protons is of the order of the dimensions of typical field irregularities (30 pc) at 10^{17} eV, of the order of the half thickness of the disc (300 pc) at 10^{18} eV and comparable to the distance between adjacent Galactic arms (3 kpc) at 10^{19} eV. Since Galactic source candidates (supernovae, supernova remnants, pulsars) are no longer many Larmor radii away at these energies, it is reasonable to expect an increasingly anisotropic distribution for the Galactic component of the cosmic rays, and one might even hope to identify individual sources at the highest energies. The distances of extragalactic source candidates (extragalactic supernovae, rich clusters of galaxies, radio and Seyfert galaxies, quasars, intensive x-ray sources) are several orders of magnitude larger, but the magnetic field in intergalactic space might be proportionally weaker and thus individual sources might still emerge.

For nuclei of given total energy the Larmor radius r_L is proportional to Z^{-1}, thus for heavier nuclei the distribution is more isotropic and individual sources are more difficult to see. If, on the other hand, some of the primaries are neutral (n, γ, ν), then there should be some sharp peaks in the actual distribution of arrival directions, although in the

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observed distribution these peaks would be broadened by errors in the measured directions of primaries (typically five degrees). Such neutral primaries would be those generated close to the regions where the initial charged particles (heavy nuclei or protons) were accelerated; neutral particles may also be produced along the trajectories of charged particles.

In the present paper we concentrate on the large-scale features, ie on apparent deviations from isotropy on much larger scales than the uncertainty of the individual arrival directions. The energies will be mostly restricted to $E > 10^{17}$ eV with particular emphasis on the highest energies $E > 10^{19}$ eV. It is in this latter region that a huge anisotropy has recently been proposed by Krasilnikov (1974) and some further support has been given to that proposal by a joint letter of the Leeds, Volcano Ranch and Yakutsk groups (Krasilnikov *et al* 1974). The statistical significance and possible physical implications of these results will be discussed in detail.

In the second part of our search for anisotropies (Kiraly *et al* 1975) small-scale features will be examined, ie excesses of intensity within small neighbourhoods of specific point source candidates (pulsars, supernova remnants, extragalactic supernovae, x- and γ-ray sources, radio galaxies, Seyfert galaxies and quasars). Such small-scale anisotropies might be expected to exist for neutral primaries at all energies but for charged primaries at the very highest energies only.

2. The present experimental situation

2.1. Usual tests of isotropy

Statistical evidence for anisotropy can be obtained by first assuming complete isotropy and then calculating the probability for a chance fluctuation of given type that produces at least as large an apparent anisotropy as that observed. The resulting probability is somewhat loosely called the chance probability or the significance level of the observation. The smaller this chance probability is the more confident one can be that the original hypothesis of isotropy was false. This latter statement, however, is only true if the measure adopted for the apparent anisotropy has at least some physical relevance. The most important methods are the following:

(*a*) Harmonic analysis (usually only first and second harmonics are used).

(*b*) χ^2 tests.

(*c*) Extreme deviations from the average.

(*d*) Correlation with theoretical expectations.

Method (*a*) is the obvious choice at lower energies, where low-order spherical harmonics are expected to predominate (see eg Davies 1954). Since the distribution in declination is strongly influenced by somewhat uncertain observational effects, an analysis in terms of spherical harmonics is usually not feasible. The harmonic analysis is then carried out in right ascension (RA) only, including either all showers in the given energy region or those in certain declination (δ) bins only. Most of the published results have been obtained by the first method. Significance levels are usually based on the distribution of amplitudes, although sometimes the phase information is also explicitly used. A change of phase with declination can give some information about the spherical harmonics involved.

Whereas the harmonic analysis is especially suited to very large-scale anisotropies, χ^2-type tests are much more versatile and can be almost equally well used in searches for large- or small-scale features, depending on the size of the RA–δ bins. Since the power of this method depends much less on the relative position of the intensity peaks than is the case with the harmonic analysis, it is preferable at the highest energies, where the angular distribution might have a complicated shape. The only complication arises because non-overlapping large bins involve some selection effects, but this difficulty can be avoided by taking overlapping cells. In that case, however, the distribution of χ^2 has to be worked out by Monte Carlo (MC) methods. Because of this difficulty, published results usually refer to small cells (eg 10° in δ times 15° in RA).

A search for high maxima in small bins is a powerful method for the detection of a small number of fairly intensive point sources, while larger bins are better for extended sources. There might also be angular distributions in which low minima or large values of the range (maximum – minimum) give a more distinctive feature, although these are physically somewhat less likely.

Method (*d*) is as good as the underlying theories. It has been widely applied and has the advantage that even negative results are very useful in rejecting some specific source distribution and/or propagation models.

2.2. *Energies below 10^{19} eV*

In the fifties and early sixties several marginally significant harmonics were reported at energies below a few times 10^{17} eV. Most of the results of these early experiments have been compiled by Delvaille *et al* (1959) and later in greater detail by Sakakibara (1965). It is an interesting question as to whether the whole set of these data gives much stronger evidence for an anisotropy than do the individual results. The question was answered in the negative by Delvaille *et al* (1959), the main argument being a 'publication effect': more significant amplitudes have a higher probability of being published. Daily and seasonal changes of the atmospheric conditions might also have given rise to some spurious sidereal waves. These conclusions were contested by Sakakibara (1965) on the grounds of the consistency in the phases of the measured first harmonics over a wide range of energy. An interesting feature was that the sidereal time of the maximum seemed to switch over from 22 h to 10 h at 5×10^{15} eV. The probability of a purely statistical explanation for the whole phenomenon was claimed to be very low ($<0{\cdot}1\,\%$). We shall return to this argument in § 3.1.

With the advent of giant air shower arrays having detection areas as large as several tens of square kilometres there has been a fast increase in the detected number of showers with primary energies above 10^{17} eV. The present world statistics is several tens of thousands for $10^{17}\,\mathrm{eV} < E < 10^{18}\,\mathrm{eV}$ and well above one thousand for $10^{18}\,\mathrm{eV} < E < 10^{19}\,\mathrm{eV}$. The above numbers refer to those showers with zenith angles and core distances within specified limits so that both the arrival directions and the primary energies are relatively well known. Detailed discussions of these data from the point of view of anisotropy have been published, eg, in the Proceedings of the International Cosmic Ray Conferences: the Volcano Ranch data have been summarized by Linsley (1963), while the latest results of the Haverah Park and Sydney groups have been given by Lapikens *et al* (1971), Brownlee *et al* (1973) and by Brownlee *et al* (1970), Bell *et al* (1971), and Bell *et al* (1973) respectively.

The findings of all these groups can be summarized in the simple statement that there seems to be no indication for large-scale anisotropies between 10^{17} and 10^{19} eV. Fairly stringent upper limits have been found for the first harmonic amplitudes (Lapikens *et al* 1971). χ^2 probabilities for uniformity in RA are not particularly low in any of the

10° declination bins covered by the Sydney array (Bell *et al* 1973). High intensities in individual bins proved to be random fluctuations after further data had been accumulated (Lapikens *et al* 1971). Several suspected source regions have been checked for excesses but no excess has been found (Brownlee *et al* 1973, Bell *et al* 1971, Bell *et al* 1973).

2.3. Energies above 10^{19} eV

Until the last few months there seemed to be no evidence for any large-scale anisotropy at the highest energies either (see Linsley and Watson 1974, and the references given in § 2.2), with the possible exception of the newest Sydney data (Bell *et al* 1973) which, according to the authors, were 'not obviously isotropic'.

In September 1974 a huge first-harmonic-type anisotropy was proposed in the northern hemisphere by Krasilnikov (1974), the evidence being based mainly on the Yakutsk and Haverah Park data. This work has been followed up in a joint letter by the Yakutsk, Leeds and Volcano Ranch groups (Krasilnikov *et al* 1974), giving strong support for a first harmonic in RA above $\delta = +30°$ and for an even more significant second harmonic below $\delta = -30°$. The harmonic analysis was carried out by first grouping the data into 2 h bins in RA. The amplitudes and phases of the first and second harmonics were given as 64·6%, 13·5 h and 100·6%, 7·8 h and 19·8 h respectively, with chance probabilities 7×10^{-3} and 2×10^{-3} respectively. However, in spite of the apparently overwhelming evidence the results were not regarded by the authors as completely conclusive.

In view of the great importance of any positive evidence for a genuine anisotropy we have decided to re-analyse the data from various aspects. The results, as will be shown in § 3.2, give some further support to doubts about the significance of the anisotropy.

The 119 shower directions used in our analysis are those used by Krasilnikov *et al* (1974), apart from any small differences due to reading-off errors in the 20 Yakutsk showers (Krasilnikov 1974), in the 50 Sydney showers (Bell *et al* 1973) and in the 3 Cornell showers (Linsley and Watson 1974). For the 14 Volcano Ranch and 32 Haverah Park showers the directions used have been obtained from J Linsley and A A Watson (private communication). The distribution of showers in celestial coordinates is given in figure 1 (equal area Aitoff projection). Some relevant directions of the Galaxy and supercluster are shown as well. The most conspicuous regions of high apparent intensity have been shown as circles on the celestial sphere. The central directions and radii of these peaks are given in table 1. Beside the usual celestial and Galactic coordinates the so called supergalactic coordinates of the central directions have also been included. The supercluster, defined by de Vaucouleurs (1958) contains about 10^4 galaxies in a flattened system centred on the Virgo cluster (supergalactic coordinates $b^\circ \simeq 0°, l^\circ \simeq 100°$). The Galaxy is near to the edge of the system, the plane of which is the plane $b^\circ = 0$.

3. The statistical significance of the proposed anisotropies

3.1. Statistical selection effects

It often occurs in experimental research that the collection of further data is very time consuming or expensive and one is compelled both to formulate and to check a hypothesis on the basis of the same set of data. Of course this statement should be qualified, because in proposing a new hypothesis one always makes use of some intuition or

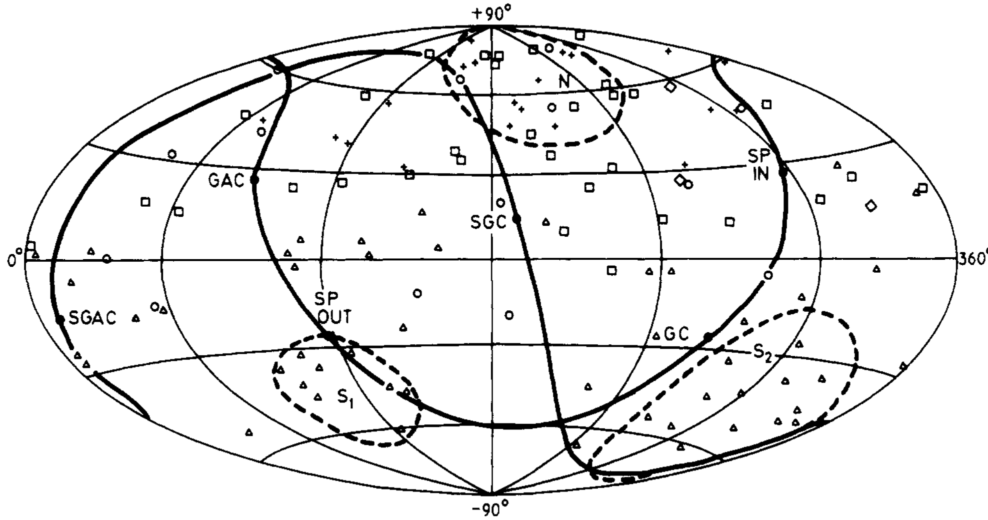

Figure 1. Aitoff equal area projection of the arrival directions of 119 showers with primary energies above 10^{19} eV (celestial coordinates). The regions of high apparent intensity are tentatively given as circles on the celestial sphere (N, S_1 and S_2). The Galactic and supergalactic equators are represented by full curves. Abbreviations: GC and GAC Galactic centre and anticentre, SP IN and SP OUT inward and outward spiral directions, SGC and SGAC supergalactic centre and anticentre. Symbols for the detecting stations: ◇ Cornell, □ Haverah Park, △ Sydney, ○ Volcano Ranch and + Yakutsk.

Table 1. The directions of the intensity peaks in celestial, Galactic and supergalactic coordinates.

Peak	RA (deg)	δ (deg)	l'' (deg)	b'' (deg)	l^0 (deg)	b^0 (deg)	Radius (deg)	Number of observed showers
N	210	65	111	51	53	21	25	21
S_1	112·5	−45	258	−12	195	−59	20	9
S_2	305	−45	355	−35	224	27	30	13

theory based on previous experience. The degree of significance is often overestimated in such situations. While sometimes it is almost a trivial problem to revalue these 'inflated' significance levels, in other cases the procedure is difficult and somewhat equivocal. Where the result is of great physical importance a very close scrutiny is obviously justified. The best method seems to be to simulate statistically some of the selection processes leading to the hypothesis and to correct the significance levels accordingly.

As an illustration (and also because of its relevance to our present subject) we take the results of Sakakibara (1965), which have already been mentioned in § 2.1. On the basis of a review of the results on first sidereal harmonics T_{max} was given by Sakakibara for 43 marginally significant waves in the energy region 5×10^{13} eV $< E < 5\times10^{17}$ eV. She then noticed that most of these T_{max} values were in the (22 ± 6) h bin below 5×10^{15} eV and in the (10 ± 6) h bin above that energy. The actual numbers in the four bins (taking the border-line cases with half weight) were 20·5 and 6·5 for the low-energy region while

4 and 12 at higher energies. A χ^2 probability was calculated with the assumption that the expected ratio of the numbers in the (22 ± 6) h and (10 ± 6) h bins is energy independent (ie complete isotropy was not assumed), and this assumption was rejected on a 0·1 % level. The conclusion was that the cosmic rays are anisotropic in the whole energy region and there is a change of phase at 5×10^{15} eV.

It should be stressed that there appeared to be no *a priori* reason for choosing exactly those separating lines in energy and in sidereal time. We consider that any energy between 10^{15} and 10^{16} eV and any sidereal time (integral hours) might have served as separating lines for the four bins, always assuming that both regions in sidereal time are 12 h wide. By calculating the maxima of the χ^2 values for all these possibilities both in the actual case and for random sets one obtains a much more realistic estimate of the significance. In fact, out of 5000 random (ie completely 'isotropic') sets of 43 $T_{\max}$ values about 6% gave higher maximized χ^2 than the actual measurements. This result greatly weakens the evidence for a change of phase and for anisotropy in general. The evidence is somewhat further weakened by the possibility of some other 'plausible' patterns for the change of phase with energy (linear change, two changes, etc). Of course there are further plausibility arguments both for and against the change in phase being genuine. A gradual change of phase between 10^{15} and 10^{16} eV has indeed some recent theoretical justification (Bell *et al* 1974), although the lack of significant amplitudes between 10^{17} and 10^{19} eV is hard to reconcile with the model. A recent, very carefully analysed experimental result also suggests some anisotropy at 6×10^{13} eV (Gombosi *et al* 1975), but the phase does not fit in with Sakakibara's suggestions. For the earlier results there is also the possibility of a psychological selection effect related to that suggested by Delvaille *et al* (1959): it might well happen that marginally significant results have a somewhat higher probability of being published if the phases are in good agreement with previous results at the same energies. Finally, if the corrections for the atmospheric effects are not done carefully enough, then the interference of diurnal and seasonal variations of the atmospheric conditions might also give rise to some correlation of sidereal maxima, and this effect should be more important at lower energies where the measured amplitudes are smaller.

Summarizing, selection effects cannot be completely eliminated, but a MC simulation of the selection process might give fairly reasonable confidence levels. However, the final decision on the fate of a hypothesis is always made on the basis of new, independent data.

3.2. The evidence for the proposed anisotropy above 10^{19} eV

The low chance probabilities for the observed first and second harmonics above $\delta = 30°$ and below $\delta = -30°$ respectively give no straightforward answer to the question of whether the effect is likely to be genuine. There are at least two selection effects involved: the selection of the harmonic method and of the declination intervals. We consider that there are no strong *a priori* arguments for either of them. In fact, if one assumes a rapidly converging series of spherical harmonics which is the usual prerequisite of giving preference to the harmonic method, then the anisotropy is expected to be most important at low declinations. If the dominant term is, for example, the first spherical harmonic, representing a net streaming of the cosmic ray gas, then the amplitude of the first harmonic in RA should vary as $\sin \chi \cos \delta$, where χ is the angle between the directions of the net flow and the axis of rotation of the earth (Bell *et al* 1974). Thus the maximum amplitude is expected at $\delta = 0$, whatever the value of χ. Since, however, this argument is not directly applicable to second harmonics and the change of coverage with declination might also somewhat favour higher latitudes, the check of the harmonic analysis will be carried out without giving preference to low latitude results. If, on the other hand, the deviation from isotropy is caused by fairly narrow peaks, as a simple inspection of figure 1 might suggest, then methods (*b*) and (*c*) discussed in § 2.1 are *a priori* preferable.

The selection effect due to the choice of the declination intervals can be reduced by a detailed study of the actual and random distribution of the harmonic components in several declination bands of different widths. The effect of using the harmonic method can be checked by comparing the significances with those obtained by methods (*b*) and (*c*). Finally, method (*d*) checks the physical plausibility of the proposed anisotropy.

The tests described below are somewhat sophisticated versions of the methods (*a*)–(*d*) discussed in § 2.1. Whenever the statistical distributions have proved to be too complicated for analytical treatment, MC calculations have been carried out with at least 1000 sets of 119 arrival directions. Since the declination dependence of the observational coverage is somewhat uncertain, all tests have been restricted to check the uniformity in RA. In view of the daily scanning of the whole RA range observational biases are not likely to have any important effect.

(*a*) The harmonic analysis has been carried out for a large number of declination bins of different lengths and the chance probabilities of the most 'significant' single harmonic amplitudes have been compared for the observed and for the MC sets. The RA values have not been grouped into bins when calculating the harmonic components.

First, the arrival directions have been grouped into 20 declination bins containing 6 showers each (except for the first, ie northermost bin, that contained 5). The amplitudes and phases of the first and second harmonic components as well as the median declinations of these basic groups are given in figure 2 for the observed showers. The harmonic analysis was then carried out for larger declination bins containing 2,3 etc consecutive basic groups, ie altogether for 190 bins containing at least 11 showers each. The 'significance levels' of the first and second harmonic amplitudes were calculated as $p(>a) = \exp(-a^2n/4)$, where a is the amplitude and n is the number of showers in the given declination bin (Krasilnikov *et al* 1974). For the observed case the most significant first harmonic was in the first 29 showers ($\delta > 48°$, $a = 0{\cdot}88$, $T_{max} = 13{\cdot}9$ h, $p(>a) = 3{\cdot}5 \times 10^{-3}$), while the most significant second harmonic was in the last 24 showers ($\delta < -30°$, $a = 1{\cdot}05$, $T_{max} = 7{\cdot}8$ and $19{\cdot}8$ h, $p(>a) = 1{\cdot}3 \times 10^{-3}$). 2000 random sets of 119 showers have been similarly analysed and in 8·3 % of the cases $p(>a)$ was less than $1{\cdot}3 \times 10^{-3}$ for either a first or a second harmonic. Even when the analysis was restricted to declination intervals containing at least 23 showers (the smallest interval used by Krasilnikov *et al*), this percentage was only reduced to between 4 and 5 %, which in our opinion is still too high to be considered more than an indication for anisotropy.

The evidence is somewhat further weakened by the fact that the shower directions previously published by the Sydney group (Brownlee *et al* 1970, Brownlee 1970) give much less significant second harmonics for $\delta < -30°$ The difference is presumably mainly due to the different threshold energies ($1{\cdot}5 \times 10^{19}$ eV for the new data, 10^{19} eV for the earlier results). It seems to be unlikely that a genuine peak should be so strongly affected by changes of threshold which are comparable to energy uncertainties.

(*b*) For the χ^2-type tests of uniformity the basic declination bins contained 12 showers (11 for the first) and the RA bins were 2 h wide, ie there were 120 basic cells altogether. Larger cells with double and treble sizes in both directions were constructed from 4 and 9 basic cells respectively. These larger cells overlapped in both directions, so we had

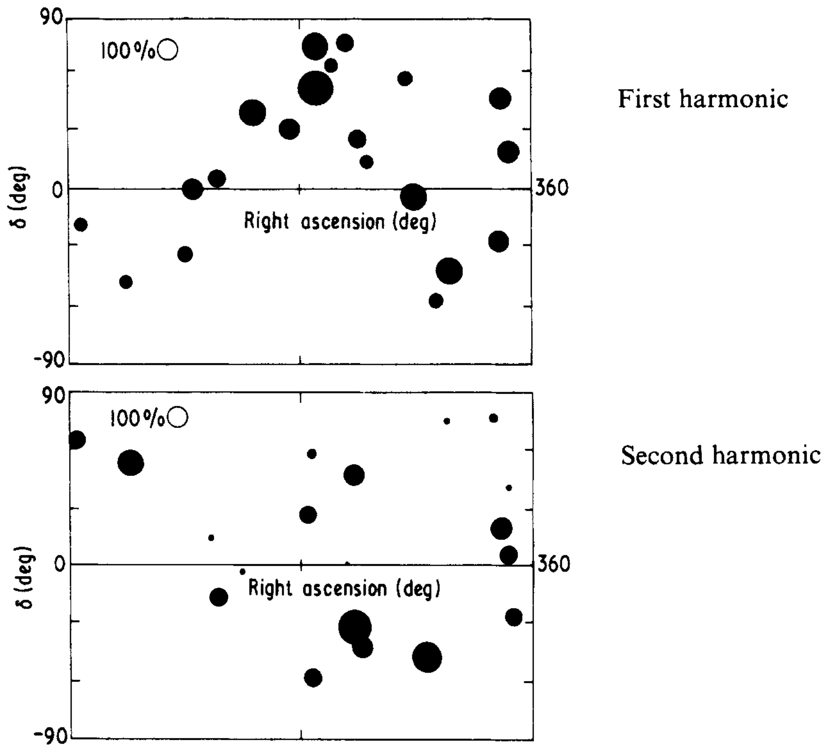

Figure 2. Phases and amplitudes of the 'basic' groups containing 6 showers each (except for the northernmost group that contains 5). The coordinates of the centre of each circle give the median declination of the group and the RA belonging to the maximum respectively, while the radius is proportional to the amplitude. Amplitudes above 100% represent closer grouping in RA than expected for a pure harmonic. For the extreme case of the complete coincidence of all RA values in a group one would have 200%.

108 double and 96 treble cells. Denoting by n_{ijk} the number of showers in the ith δ, kth RA interval for size i (1 for basic, 2 for double, 3 for treble cells), χ_i^2 was calculated as

$$\chi_i^2 = \sum_{j,k} \frac{(n_{ijk} - \langle n_{ijk} \rangle)^2}{\langle n_{ijk} \rangle}.$$

An analytical calculation of the random distribution of χ_i^2 is somewhat complicated because of the overlaps, but it is fairly easy by MC simulation. The chance probabilities for the observed showers are given in table 2.

Table 2. Significance levels obtained for χ^2, maxima, minima and ranges.

Cell type	χ^2	$p(\geqslant\chi^2)\%$	max	$p(\geqslant\max)\%$	min	$p(\leqslant\min)\%$	R	$p(\geqslant R)\%$
Basic	103	68·7	3	100	0	100	3	100
Double	93	41·0	8	95·8	0	71·9	8	85·5
Treble	79	32·4	16	40·6	2	18·9	14	21·2

(*c*) Maxima (max), Minima (min) and ranges (R = max − min) have also been selected from among the entries of each of the above three sets of cells. The chance probabilities given in table 2 indicate the fractions of MC results which gave maxima and ranges larger than or equal to those observed and minima smaller than or equal to those observed. Minima for the two smaller cell sizes are mostly zero and therefore not very informative.

(*d*) Expected and observed numbers of showers in some meaningful directions are given in table 3. The expected numbers were again calculated on the basis of randomness in RA. The method will be detailed in the appendix.

Table 3. Observed and expected numbers of showers in some relevant regions of the Galaxy and supercluster.

Region		Galaxy			Supercluster		
		Observed	Expected	(obs − exp)/σ	Observed	Expected	(obs − exp)/σ
±30° band around equator		50	61·2	−2·1	63	64·5	−0·3
Northern hemisphere		67	69·2	−0·5	75	66·7	+1·6
Around centre	30° circle	8	6·6	+0·6	5	7·1	−0·9
	60° circle	25	23·5	+0·4	30	29·0	+0·2
	90° circle	49	49·5	−0·1	66	65·0	+0·2
Around anticentre	30° circle	7	8·5	−0·6	7	5·9	+0·5
	60° circle	29	34·5	−1·2	25	25·6	−0·2

The chance probabilities obtained in tests (*a*)–(*c*) suggest that the observed set of showers is not very different from a random distribution from most points of view. However, these tests are somewhat arbitrary and might be suspected to be biased. Therefore, we have decided to carry out an additional check based more on the experience of physicists than on exact statistics. Completely random two-dimensional distributions of 119 points have been generated together with the distribution of the actual shower directions transformed to uniform coverage in δ. The coverage effects have been eliminated by using random numbers as declinations, and the right ascensions have been assigned to these declinations in the same order as observed, ie the same RA has been assigned to the highest δ in both cases etc. The printouts of the observed set and of nine random sets have been shown to several physicists working in related fields. No one could pick out the observed set as unusually anisotropic. The first four printouts are presented in figure 3. Each plot contains 40 rows and 48 columns; a '2' means two showers in the same bin. The observed set has been printed upside down in order to prevent a selection by memory rather than by the comparison of the distributions.

As we see, two-dimensional random point distributions abound in patterns catching the eye and proving quite significant when analysed by methods more or less tailored to suit them. The intensity 'peaks' in the observed distribution are therefore interpreted as an indication only for a possible anisotropy. The role of such indications is in providing a datum with which future results can be compared and also in providing more stringent limits on certain types of proposed anisotropies.

There might be some supporting evidence for a genuine anisotropy in the correlation of T_{max} below and above 10^{19} eV as suggested by Krasilnikov *et al* (1974), but the huge differences in amplitude seem to weaken the argument. A direct connection with the anisotropy suggested by Sakakibara (1965) is very difficult to visualize.

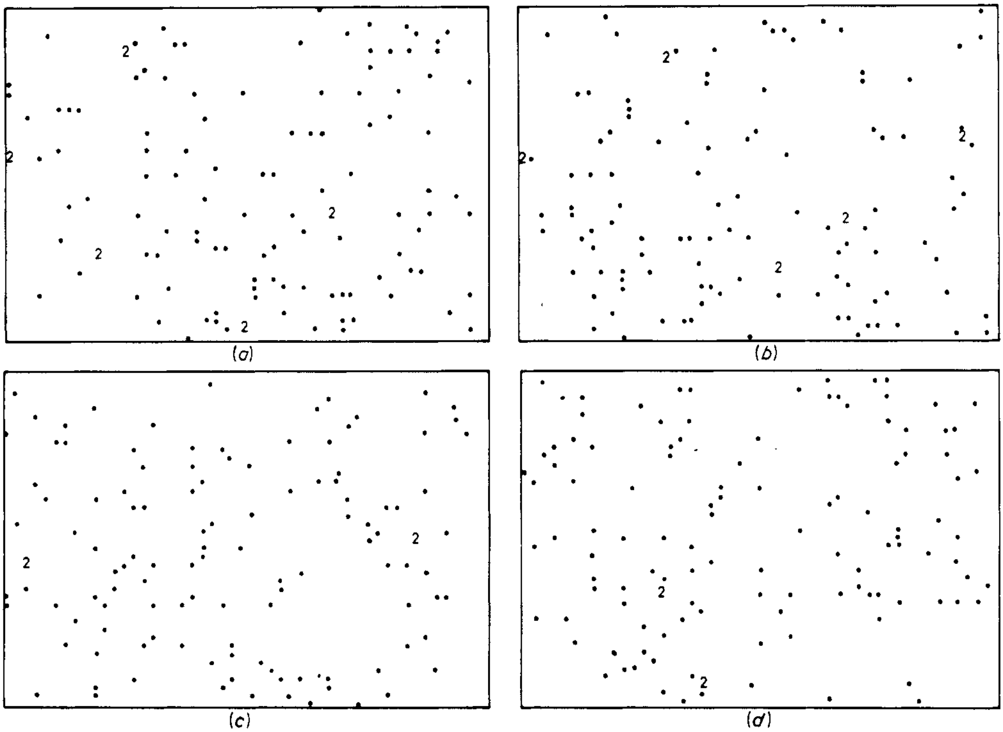

Figure 3. (*a*) A 'statistically equal coverage' representation of the observed set of showers compared with (*b*)–(*d*) three random sets. The observed set has been plotted upside down in order to reduce 'memory effects'. The RA values are grouped into 48 bins, the declinations into 40 bins. A '2' represents two points in the same RA–δ bin.

4. Discussion and conclusions

Although the statistical evidence for a genuine anisotropy has been found to be weak, it seems to us justified to have a closer look at some possible implications. At first we shall present some tentative ideas about the origin of the three apparent intensity peaks (figure 1, table 1), then a brief discussion will be given of the results summarized in table 3.

The peaks might represent, in order of decreasing plausibility:

(i) statistical fluctuations;

(ii) independent contributions from three intensive sources superimposed on a more or less isotropic background;

(iii) preferential directions reflecting more the configuration of the magnetic field than that of the sources.

Hypothesis (i) has already been discussed in detail. Of course it does not imply the physically unlikely assumption of a complete isotropy, but only that genuine intensity enhancements have much smaller amplitudes than the apparent ones and also the patterns are completely unrelated.

For hypothesis (ii), the source directions are assumed to be inside the perimeters of the peaks. The spread would then be due to small deflections in weak randomly directed magnetic fields. In this case high energies should show closer grouping, but the available data (Linsley and Watson 1974, Bell *et al* 1973) do not give support, except perhaps for the peak S_2, where higher energies (smaller serial numbers in Bell *et al* 1973) seem to be concentrated into one half of the peak. If this grouping reflects a genuine effect, then the source should be farther away from the Galactic plane than the centre of the peak. Some additional evidence for or against hypothesis (ii) might be obtained from the north–south asymmetry observed at different stations, because the peaks should represent maxima in δ as well as in RA. Since the peaks show no correlation with either the Galactic or the supergalactic (de Vaucouleurs 1958) plane, this hypothesis favours a universal origin. In that case, the peaks would correspond to three intensive sources several hundred megaparsecs away, while the background would come from more distant unresolved sources. Although the intergalactic fields should be very weak because of the small angular spreads, the time delay of the particles would still be of the order of 10^7 yr which might explain why we cannot see the electromagnetic radiation from the sources. The lifetime and space density of the quasars might fit in with this conclusion.

Hypothesis (iii) is even more uncertain because of the infinite possibilities for the field configurations. The experimental evidence for the fields is very vague outside our local neighbourhood of about 2 kpc in the Galactic plane. The local field points approximately in the direction of the inward spiral arm, although irregularities make the interpretation somewhat uncertain (Berge and Seielstad 1967, Manchester 1974, Vallée and Kronberg 1973). One might, of course, invent some complicated fields by which the particles coming from the Virgo cluster (or even from the centre of the Galaxy) would be channelled into the observed directions, but in our opinion the evidence is by far not enough for such speculations. For conventional models of the Galactic field it was shown by Karakula *et al* (1972) and by Osborne *et al* (1973) that the observed angular distributions at somewhat lower energies do not fit in with the hypothesis that the primaries are protons produced in the Galaxy.

The agreement between the observed and statistically expected intensities given in table 3 is quite good. There is only a single deviation above 2σ out of 14 data, which is roughly what one expects. Thus there seems to be no evidence for any of the physically expected anisotropies. The observed $2{\cdot}1\sigma$ deficiency on the other hand puts very stringent upper limits on any genuine enhancement connected with the Galactic plane. Although this does not exclude a Galactic origin altogether, the magnetic field model (including some halo component) and the charge composition of the primaries should be rather cleverly chosen in order to avoid a discrepancy.

In the case of a universal origin one might expect some enhancement from the direction of the plane of the supercluster in general and from the Virgo cluster in particular (Strong *et al* 1974). The magnitude of the expected enhancement, however, is rather uncertain, and we do not think that the results in table 3 exclude this hypothesis. It might also happen that the space density of the sources is so small that there is no source in the supercluster.

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Appendix

The 'statistical expectation' of the number of showers coming from a given region G of the sky has been calculated in the following way. For each observed shower one draws a line of constant δ (see figure 4) and calculates the ratio of the section inside G to the total length (360° if RA is measured in degrees). If one assumes complete isotropy (or at least no change in intensity for constant δ), then these ratios can be interpreted as the probabilities p_i of the ith shower falling inside G. The change of observational coverage with δ does not affect these probabilities.

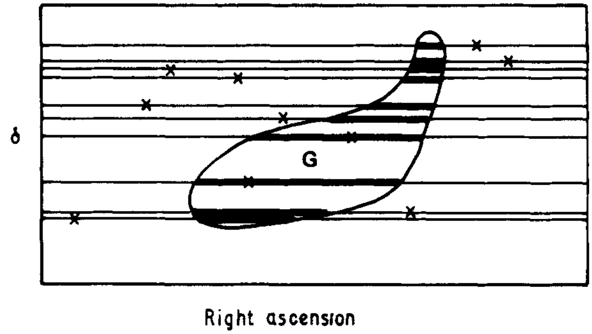

Figure 4. The contributions of the observed showers to the expectation inside G.

The statistical expectation for the number of showers n which fall inside G is then:

$$\langle n \rangle = \sum_{i=1}^{N} p_i,$$

where N is the number of those observed showers for which the constant δ line intersects with G.

The standard deviation of n is also easy to calculate. Introducing the notations

$$\bar{p} = \frac{1}{N} \sum_{i=1}^{N} p_i, \qquad \delta p_i = p_i - \bar{p}$$

we have

$$\sigma^2 = \langle (n - \langle n \rangle)^2 \rangle = \sum_{i=1}^{N} p_i(1-p_i) = \sum_{i=1}^{N} (\bar{p} + \delta p_i)(1 - \bar{p} - \delta p_i) = N\bar{p}(1-\bar{p}) - \sum_{i=1}^{N} (\delta p_i)^2.$$

Thus the standard deviation of n is always smaller than it would be for a binomial distribution with the same N and $p = \bar{p}$, except in the limiting case $p_i = p$ for all values of i. It is also easy to see that shower lines being completely inside G give no contribution to σ^2. If a Gaussian approximation is unacceptable for the significance limits, MC methods should be used.

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